A low-dimensional model of separation bubbles

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Workshop on Bifurcation Analysis and its Applications

Montreal, July 7-10, 2010

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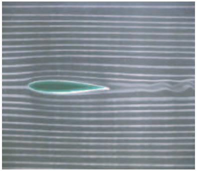
  - suggestion of a number of non-trivial questions to be answered experimentally;
  - model based on intuitive physical variables.

Introduction

What is the separation[a]?



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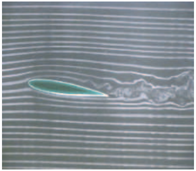

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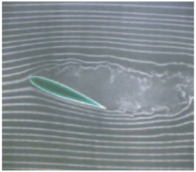

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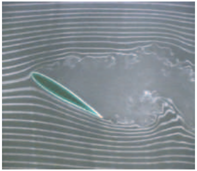

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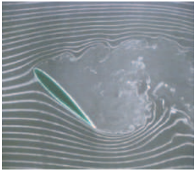

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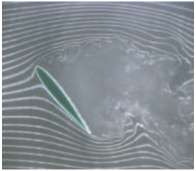

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[a] Multimedia Fluid Mechanics, Homsy *et al.* (2001)

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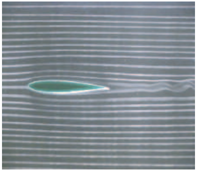

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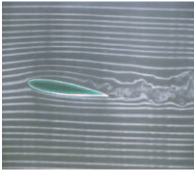

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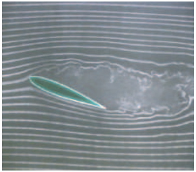

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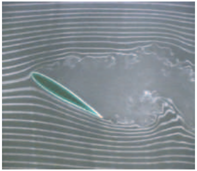

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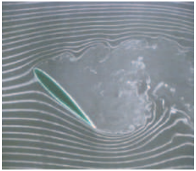

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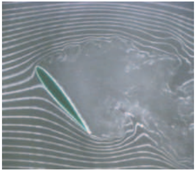

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Historical remark: term "separation bubble" is due to Jones (1933).

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  - *low-dim*, for computational efficiency in real flight;
  - *physically motivated*, to reflect actual behavior.

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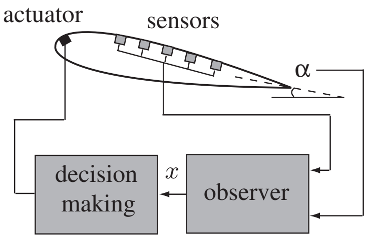
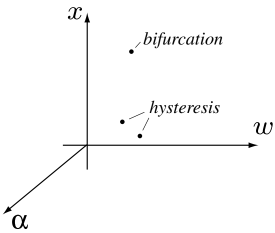
Closed-loop dynamic control system



(a) Feedback control.



(b) State space.

Figure: The key dynamic elements—bifurcation and hysteresis—to be captured by the minimal number of parameters, namely the bubble size x, the angle of attack α, and the actuation amplitude w.

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Approaches to low-dimensional modeling

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  Disadvantages: (a) unreliable for open flows, (b) physical mechanisms remain uncovered, (c) need a full solution.

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Classical example of the successful phenomenology: Landau equation (Landau, 1944; Stuart, 1960):

$$\frac{\mathrm{d}A}{\mathrm{d}t} = A - \gamma A\,|A|^2\,.$$

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State-of-the-art low dimensional model[a]

- Physical variables:
  (i) lift Z
  (ii) separation state $B = \begin{cases} 0, \text{fully attached} \\ 1, \text{fully separated} \end{cases}$

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$$B_{tt} = -k_1 B_t + k_2 \left[B_s(\alpha) - B \right],$$
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Question: is this linear model adequate?

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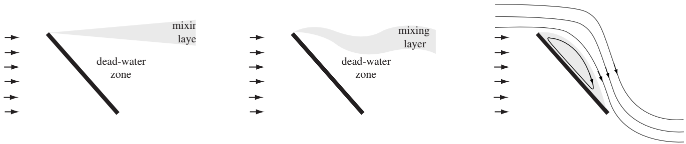
- *Mechanism*: the excitation (vs. forcing) generates Large Coherent Structures transferring high momentum fluid towards the surface:



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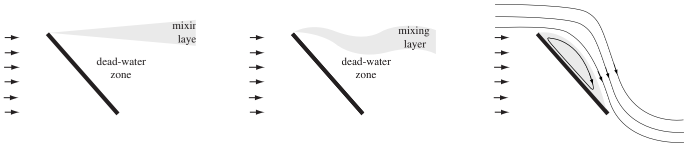

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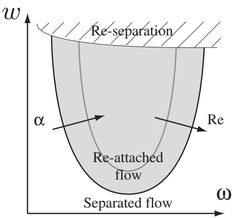
- *Threshold for actuation* to achieve reattachment and *effects of amplitude w and frequency ω of actuation on bubble size x* (Nishri & Wygnanski, 1998)
- *Re-separation phenomena* (Krechetnikov & Lipatov, 2000)



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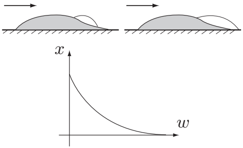
Physics of actuation (continued)

- *Primary bifurcation* in two basic experimental models:



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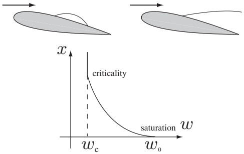

(b) Airfoil model

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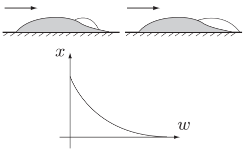

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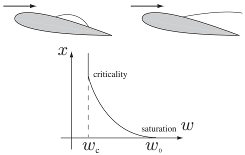

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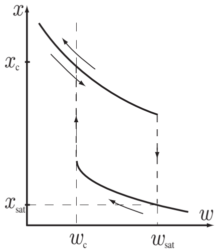
- *Hysteresis behavior* in all (α, w, ω), (Nishri & Wygnanski, 1998; Greenblatt *et al.* 2001).



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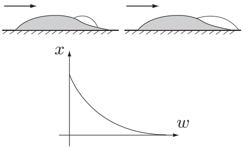

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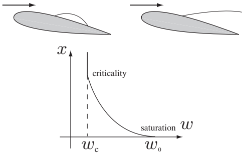

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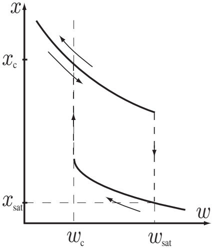
- *Hysteresis behavior* in all (α, w, ω), (Nishri & Wygnanski, 1998; Greenblatt *et al.* 2001).

- *Conclusion*: a model should be nonlinear.



Bifurcation

Motivation from real bubbles

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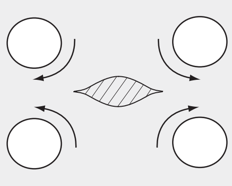

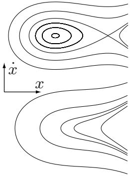

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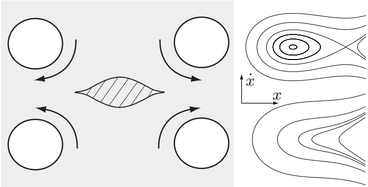

- Let x be a scalar measure of deformation from sphericity. Linear oscillation theory (Lamb, 1932) of a spherical bubble + steady state weakly nonlinear deformation theory:

$$\ddot{x} = -\mu\dot{x} + (ax - bx^2) + w,$$

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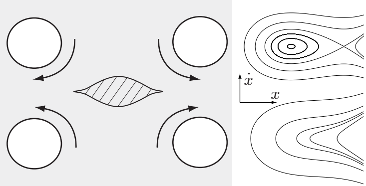

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- Bifurcation type: Takens-Bogdanov

Bifurcation

A new model: determination of variables

- Since the separation is associated with the *separation region*, it is natural to describe it with the variable representing some characteristic of a separation bubble, *e.g.* the *bubble size* x.

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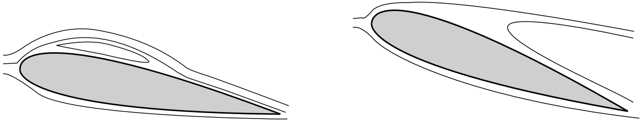

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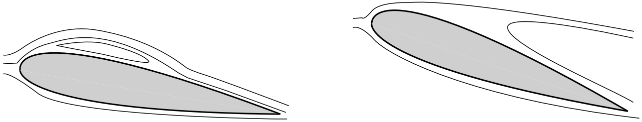

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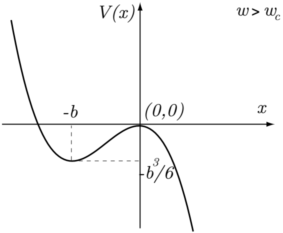
- Naturally, the bubble size $x(t; \alpha, w)$ is a function of time t, a *flight parameter*, angle of attack α, and a *control parameter* w:

$$\ddot{x} + \mu\dot{x} = F(x, w, \alpha),$$

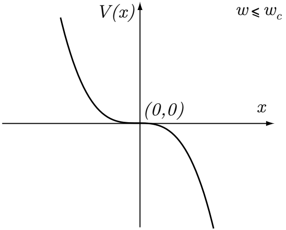
with minimal quadratic nonlinearity $F(x, w, \alpha) = x^2 + b(w, \alpha)\, x + c(w, \alpha)$.

Bifurcation

Potential function approach



(a) Potential function for a finite bubble.



(b) Potential function for an infinite bubble.

Figure: Potential function $V(x) = -\frac{x^3}{3} - b(w)\frac{x^2}{2} - c(w)x - d(w)$ with $d = 0$, $c = 0$.

Bifurcation

A new model: construction and analysis

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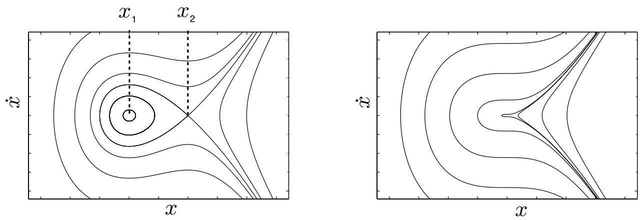
$$\ddot{x} = -\mu\dot{x} + (x-\alpha)^2 + f(w)\,x.$$



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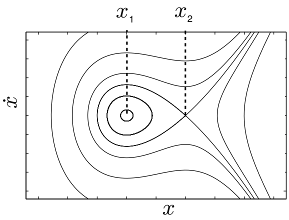

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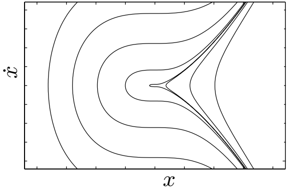

(b) uncontrolled

- Here $f(w) = a_1 w + a_2 w^2 + \dots$ represents the *nonlinear response* of the separation region to actuator excitations, for instance, of a periodic form $w = w_0 \sin \omega t$. The product $f(w)\,x$ means that the *effect of actuation depends upon the bubble size* x.

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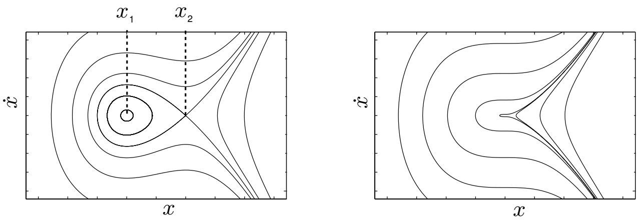
- The model is a part of the Takens-Bogdanov bifurcation:

$$\ddot{x} = -\mu\dot{x} + (x-\alpha)^2 + f(w)\,x.$$



(a) controlled (b) uncontrolled

- Here $f(w) = a_1 w + a_2 w^2 + \ldots$ represents the *nonlinear response* of the separation region to actuator excitations, for instance, of a periodic form $w = w_0 \sin \omega t$. The product $f(w)\,x$ means that the *effect of actuation depends upon the bubble size* x.
- Prediction: separation bubble should be finite-amplitude unstable.

Bifurcation
Concept of dynamic bifurcation

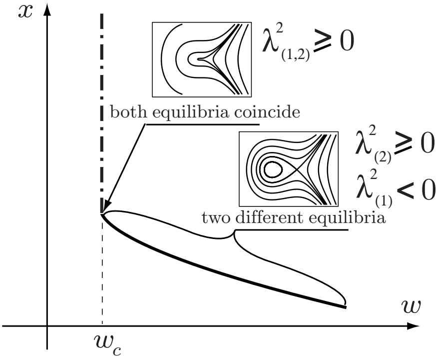
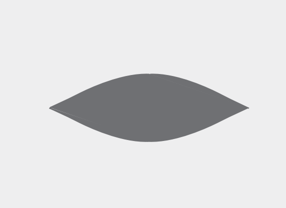
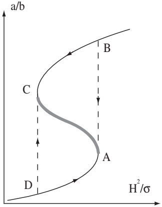


Figure: Critical curve in the (x, w)-plane: on the dynamic bifurcation; solid black line represents stable equilibria, dot-dash line is a dynamic bifurcation when bubble grows indefinitely with time. λ's are the eigenvalues of the linearization around equilibrium points.

Hysteresis
Motivation from real bubbles

- **Ferrofluid drop in a magnetic field**[a]





[a] Bacri & Salin, 1982

Hysteresis

Motivation from real bubbles

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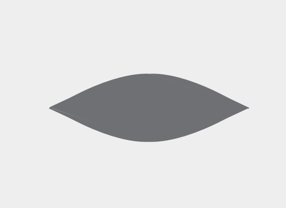

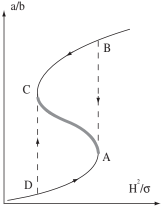

- Total energy $E_t = E_s + E_m$ is a sum of magnetic E_m and interfacial E_s contributions:

$$E_s = \sigma 2\pi a^2 e \left[e + \epsilon^{-1} \sin^{-1} \epsilon \right], \quad \epsilon = \sqrt{1 - e^2}$$

$$E_m = -\frac{VH^2}{8\pi} \frac{\mu_1}{\alpha + n}, \quad \alpha = \frac{\mu_1}{\mu_2 - \mu_1}.$$

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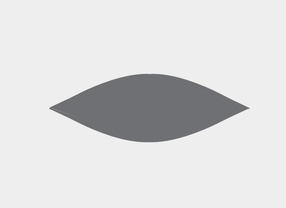

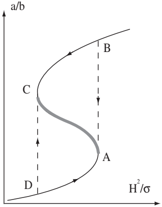

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Hysteresis
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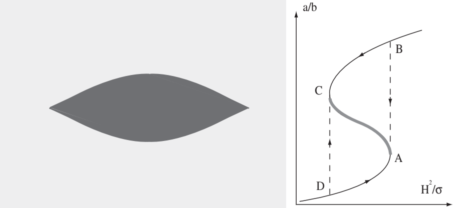
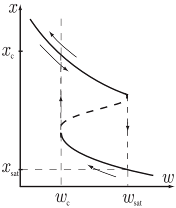


- **Conjecture**



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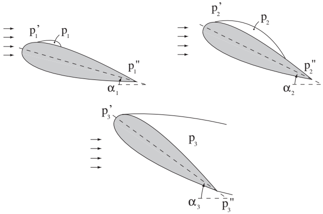
Hysteresis

Motivation: separation vs. cavitating bubble

- **Separation bubble**:



Hysteresis

Motivation: separation vs. cavitating bubble

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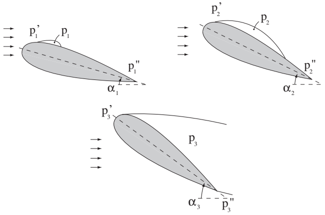

- On mechanism of separation

$$p_1' - p_1'' < p_2' - p_2'' < p_3' - p_3'',$$
$$l_1 < l_2 < l_3.$$

$p_1 > p_2 > p_3$ with $p_i < p_i'$, $i = 1, 2, 3.$

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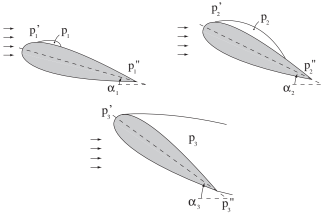

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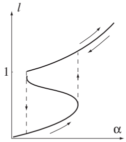
- **Cavitating bubble**:



Acosta (1955), Tulin (1953)

Hysteresis

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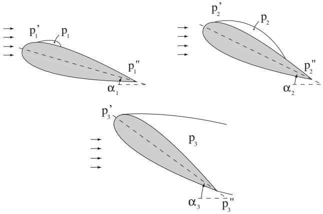

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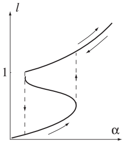


Acosta (1955), Tulin (1953)

- The behavior of a cavitation bubble is given by for partially cavitating, $l < 1$, and supercavitating, $l > 1$, foils respectively,

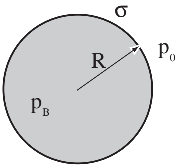
$$\frac{\chi}{2\alpha} = \frac{2 - l + 2(1-l)^{1/2}}{l^{1/2}(1-l)^{1/2}}, \quad l < 1,$$

$$\alpha\left(\frac{2}{\chi} + 1\right) = (1-l)^{1/2}, \quad l > 1,$$

Hysteresis

Motivation: static vs. cavitating bubble

- **Static bubble**:



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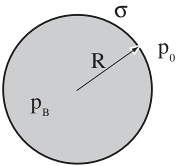

- Real static bubble behavior

$$p_B = 2\sigma/R + p_0,$$

where p_B is the pressure inside the bubble, p_0 – pressure outside the bubble, $\sigma > 0$ is the interfacial tension, and R is a radius of the bubble.

Hysteresis

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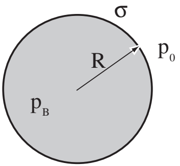

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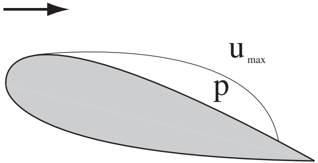
- **Cavitating hydrofoil**:



Hysteresis

Motivation: static vs. cavitating bubble

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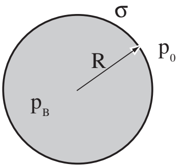

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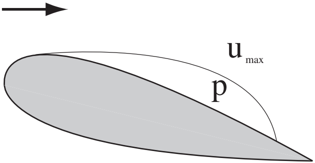


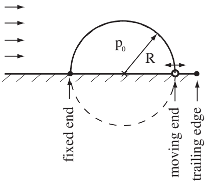
- Bubble behavior:

$$p + \rho u^2/2 = p_{\mathrm{st}},$$

where p is a dynamic pressure, and p_{st} is the pressure of a fluid at rest (at stagnation point).

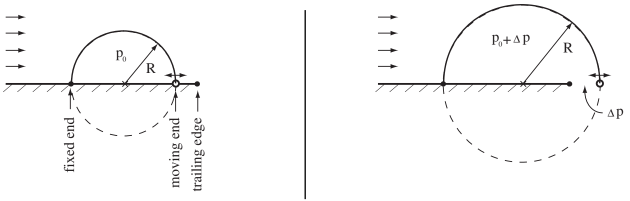
Hysteresis

Mechanical model of hysteresis: elastic bubble



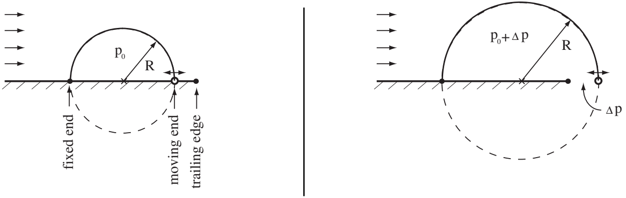
Hysteresis

Mechanical model of hysteresis: elastic bubble



Hysteresis

Mechanical model of hysteresis: elastic bubble



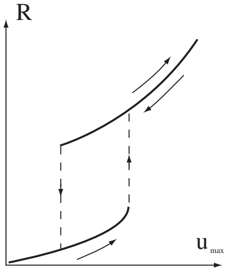
The mechanical analog of a bubble:

$$p = p_0 + \widetilde{\sigma}/R, \;\; p > p_0,$$

i.e. the bubble grows when the ambient pressure, $p = p_{\rm st} - \rho u_{\rm max}^2/2$, decreases.

$$u_{\rm max}^{\rm cr,>} \; : \;\; R_0 = \widetilde{\sigma}\left[p_{\rm st} - p_0 - \rho\left|u_{\rm max}^{\rm cr,>}\right|^2/2\right]^{-1}$$

$$u_{\rm max}^{\rm cr,<} \; : \;\; R_0 = \widetilde{\sigma}\left[p_{\rm st} - p_0 - \Delta p_0 - \rho\left|u_{\rm max}^{\rm cr,<}\right|^2/2\right]^{-1}$$



Hysteresis

Model: potential function $V(x)$ approach

Modified model:

$$\ddot{x} + \mu\dot{x} = -V_x(x; \alpha, w).$$

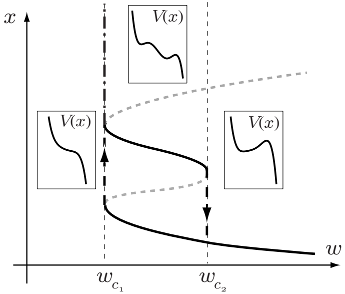


Figure: Hysteresis curve in the (x, w)-plane and corresponding potential functions; solid black lines represent stable equilibria, while dashed lines are unstable equilibria; dot-dash line represents a dynamic bifurcation (bubble size grows with time unboundedly).

Conclusions

- A new physically motivated low-dimensional model of separation bubble dynamics was constructed by contrasting and appealing to similarities with actual bubble dynamics[a]. The latter suggested
    - the proper choice of coarse variables and primary bifurcation;
    - an explaination of the nature of the hysteresis.
- Suggestions for experimental studies to improve the model:
    - investigate the finite amplitude stability of separation bubbles;
    - determine the form of the state equation for separation bubble.
- Open issues:
    - rigorous derivation of the low-dim model by *coarsening* NSEs;
    - more close connection with experimental observations and development of a *calibration procedure*.

Acknowledgements. R.K. would like to thank Prof. Anatol Roshko for stimulating discussions.

[a] Krechetnikov, Marsden, Nagib, Physica D 238, 1152 (2009)